

$$[2][b] m \frac{dv}{dt} = mg + 4\sqrt{v}$$

$$2 \frac{dv}{dt} = 2g + 4\sqrt{v}$$

$$8 \boxed{\frac{dv}{dt} = g + 2\sqrt{v}} \rightarrow \boxed{g + 2\sqrt{v} = 0} \quad 3$$

$$2 \boxed{v(0) = -16}$$

$$3 \boxed{\sqrt{v} = -\frac{g}{2} < 0} \rightarrow \boxed{\text{NO EQ SOLN}}$$

 Gravity, Fluid Resistance > 0

$$[c] m \frac{dv}{dt} = mg - 4\sqrt{v}$$

$$2 \frac{dv}{dt} = 2g - 4\sqrt{v}$$

$$6 \boxed{\frac{dv}{dt} = g - 2\sqrt{v}} \rightarrow \boxed{g - 2\sqrt{v} = 0} \quad 3$$

$$2 \boxed{v(0) = 0}$$

$$3 \boxed{v = \frac{g^2}{4}} \rightarrow \text{EQ SOLN}$$

 Fluid Resistance < 0

$$[d] \frac{dv}{dt} = 10 - 2\sqrt{v}$$

$$3 \boxed{\int \frac{1}{10-2\sqrt{v}} dv = dt}$$

$$\xrightarrow{\text{FOLD INTO } +C} 5\sqrt{v} - 5 \ln |10 - 2\sqrt{v}| = t + C \quad 6$$

$$-\sqrt{v} - 5 \ln |10 - 2\sqrt{v}| = 0 + C$$

$$3 \boxed{C = -5 \ln 10}$$

$$-\sqrt{v} - 5 \ln |10 - 2\sqrt{v}| = t - 5 \ln 10$$

$$3 \boxed{\sqrt{v} + 5 \ln |10 - 2\sqrt{v}| = 5 \ln 10 - t}$$

$$[e] f = g - 2\sqrt{v} \text{ IS NOT CONT AROUND } (t, v) = (0, 0) \quad 5$$

SINCE \sqrt{v} DNE AS $v \rightarrow 0^-$

SO E+U TELLS US NOTHING 3

$$[f] \sqrt{v} + 5 \ln |10 - 2\sqrt{v}| = 5 \ln 10 - t$$

$$\boxed{5 \ln 10 = 5 \ln 10 - t} \quad 5$$

$v = 0$ ONLY WHEN $t = 0$

$$\begin{aligned} & \int \frac{1}{10-2\sqrt{v}} dv \quad \text{LET } u = 10 - 2\sqrt{v} \quad 3 \\ & = \int -\frac{1}{2} \frac{10-u}{u} du \quad 6 \quad v = \frac{1}{4}(10-u)^2 \\ & = -\frac{1}{2} \int \left(\frac{10}{u} - 1\right) du \\ & = -\frac{1}{2} (10 \ln|u| - u) \\ & = -\frac{1}{2} (10 \ln|10 - 2\sqrt{v}| - 10 + 2\sqrt{v}) \\ & = 5 - \sqrt{v} - 5 \ln|10 - 2\sqrt{v}| \end{aligned}$$

$$[3] \quad |(2s + 2e^{2t-2s})ds + (e^{2t-2s} - 3s^2 + 6)dt = 0| \quad 9$$

P

$$P_t = 4e^{2t-2s}$$

Q

$$Q_s = -2e^{2t-2s} - 6s \quad | \quad 3$$

$$\frac{Q_s - P_t}{P} = \frac{-6e^{2t-2s} - 6s}{2s + 2e^{2t-2s}} = -3 \quad | \quad \begin{matrix} \text{FUNCTION OF ONLY } t \\ 6 \end{matrix}$$

$$\mu = e^{\int -3 dt} = |e^{-3t}| \quad 3$$

$$3 \quad |(2se^{-3t} + 2e^{-t-2s})ds + (e^{-t-2s} - 3s^2e^{-3t} + 6e^{-3t})dt = 0|$$

M

$$3 \quad |M_t = -6se^{-3t} - 2e^{-t-2s} = N_s = -2e^{-t-2s} - 6se^{-3t} \quad \text{EXACT}|$$

$$f = \int (2se^{-3t} + 2e^{-t-2s})ds$$

$$= 3 |s^2e^{-3t} - e^{-t-2s} + C(t)|$$

$$f_t = 3 |3s^2e^{-3t} + e^{-t-2s} + C'(t) = e^{-t-2s} - 3s^2e^{-3t} + 6e^{-3t}|$$

$$3 \quad |C'(t) = 6e^{-3t} \quad \text{FUNCTION OF ONLY } t|$$

$$3 \quad |C(t) = -2e^{-3t}|$$

$$3 \quad |s^2e^{-3t} - e^{-t-2s} - 2e^{-3t} = C|$$

$$3 \quad |s^2 - e^{2t-2s} - 2 = Ce^{3t}|$$

$$[4] (8xy + 2(4x^2-1)y^{-\frac{1}{2}}) - 3(4x^2-1)\frac{dy}{dx} = 0$$

$$3(4x^2-1)\frac{dy}{dx} - 8xy = 2(4x^2-1)y^{-\frac{1}{2}}$$

$$9 \boxed{\frac{dy}{dx} - \frac{8x}{3(4x^2-1)}y = \frac{2}{3}y^{-\frac{1}{2}}}$$

$$v = y^{1-\frac{1}{2}} = \boxed{y^{\frac{1}{2}}} \rightarrow \frac{dv}{dx} = \frac{3}{2}y^{\frac{1}{2}}\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2}{3}y^{\frac{1}{2}}\frac{dv}{dx}$$

$$3 \boxed{\frac{2}{3}y^{\frac{1}{2}}\frac{dv}{dx} - \frac{8x}{3(4x^2-1)}y = \frac{2}{3}y^{-\frac{1}{2}}}$$

$$\frac{dv}{dx} - \frac{4x}{4x^2-1}y^{\frac{3}{2}} = 1$$

$$6 \boxed{\frac{dv}{dx} - \frac{4x}{4x^2-1}v = 1 \quad \text{LINEAR}}$$

$$\mu = e^{\int -\frac{4x}{4x^2-1} dx} = e^{\int -\frac{1}{2}\frac{1}{u} du} = \boxed{e^{-\frac{1}{2}\ln|4x^2-1|}} = \boxed{(1-4x^2)^{-\frac{1}{2}}}$$

$3 \boxed{u = 4x^2-1}$

$$3 \boxed{(1-4x^2)^{\frac{1}{2}}\frac{dv}{dx} + 4x(1-4x^2)^{-\frac{1}{2}}v = (1-4x^2)^{-\frac{1}{2}}}$$

$$(1-4x^2)^{\frac{1}{2}}v = \int \frac{1}{\sqrt{1-4x^2}} dx + C$$

$$\begin{aligned} \frac{d}{dx}(1-4x^2)^{-\frac{1}{2}} \\ = -\frac{1}{2}(1-4x^2)^{-\frac{3}{2}}(-8x) \\ = 4x(1-4x^2)^{-\frac{3}{2}} \end{aligned}$$

$$(1-4x^2)^{\frac{1}{2}}v = \frac{1}{2}\sin^{-1}2x + C$$

$$3 \boxed{(1-4x^2)^{\frac{1}{2}}y^{\frac{3}{2}} = \frac{1}{2}\sin^{-1}2x + C} \boxed{5}$$

$$4y^3 = (1-4x^2)(\sin^{-1}2x + C)^2$$

$$3 \boxed{108 = \frac{3}{4}(\frac{\pi}{6} + C)^2}$$

$$\pm 12 = \frac{\pi}{6} + C$$

$$3 \boxed{C = -\frac{\pi}{6} \pm 12}$$

$$3 \boxed{4y^3 = (1-4x^2)(\sin^{-1}2x - \frac{\pi}{6} \pm 12)^2}$$

$$144y^3 = (1-4x^2)(6\sin^{-1}2x - \pi \pm 72)^2$$